

Threshold Policy for M/M/C/K Queue with Integrated Traffic

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Abstract

In this paper, we discuss a finite capacity queue with integrated traffic and batch services under N-policy. There are two types of customer arrive according to poisson distribution. As soon as the queue size reaches the threshold level N, the all server C are turned on and serves both types of traffic one by one upto threshold level $d (> C)$ of the customers in the system. After the threshold level d , the server provides the service of the type 1 customers in a batch whereas type 2 customers are lost. The queue size distribution is derived with the help of recursive method. Various performance measures, i.e. expected number of customers in the queue and in the system, probability of the server being turn off, under setup, busy and the expected idle/busy period etc. are determined.

Keywords: Finite queue, Integrated traffic, N-policy, Setup time, Single and Batch Services

1. Introduction

We consider M/M/C/K queueing system in which the server follows a threshold type policy with integrated traffic, bulk service including setup time. There so many kind of traffic such as voice, data and moving images are transmitted through a common resource by multiplexing them into one transmission stream, so as to utilize the link capacity efficiently. In integrated service system, different types of traffic require their own quality of service (QoS). For example, the real time traffic such as video is delay sensitive where as non real time data traffic is delay tolerant but sensitive towards error. Therefore in such type integrated communication service systems, traffic control, mechanism is required to provide the different QoS to different kind of traffic.

We dealt N-policy queueing system for two class of customer with two service modes; single and batch service modes. The optimal N-policy for a Markovian queue with multi server has been studied. The server turns off, when the system become empty and turns on whenever $N(\geq 1)$ or more customers are present in the system. Several authors have studied N-policy queueing models. N-policy controlled queueing model have potential applications in computer and communication system, where servers start processing of customers after accumulating a certain number of customers (say N customers).

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Arumuganathan and Jeya kumar [1] developed a steady state analysis of a bulk queue with multiple vacation , setup times under N-policy . Chang and Choi [2] studied performance analysis of a finite- buffer discrete time queue with bulk arrival, bulk service and vacation. Choudhary [3] considered the expected busy period for N-policy queue with general setup time. Dharmaraja et al. [4] considered phase type model for cellular networks supporting integrated voice , video and data traffic. Dudin and Klimenok [5] provided optimal admission control in a queueing system with heterogeneous traffic. Goswami et al. [6] analyzed a multi server bulk service finite-buffer queue. Hernandez Diaz , and Moreno [7] analyzed a discrete time single server queueing system with N-policy, and early setup time. Krishnamoorthy and Usha kumari [8] analyzed a queueing system with single arrival, bulk service and single departure. Tian and Zhang [9] studied a threshold vacation policy in multi server queueing system. Lee and Choi [10] gave the queueing system with multiple delay and loss priorities for ATM network. Lin and Ke [11] considered an M/M/C queueing model with C=2 in which the number of working servers can be adjusted one at a time at arrival epochs or at service completion epochs depending on the number of customers in the system.

In this paper, we propose a traffic model for a Markovian queue operating under N-policy with setup time and having two types of traffic; The inter-arrival and service time of both traffic are exponentially distributed. The paper is organized as follow. The model is described in section 2. The steady state equations are given in section 3. In section 4 we calculate the queue size distribution. Other performance indices in terms of probabilities are derived in section 5. The conclusion is given in section 6.

2. Model Description

We assume a multi server and finite capacity queueing system with setup time and state dependent arrival and service rates. The service of the customers is done exponentially in two modes i.e. single service mode and batch service mode with mean rates μ_1 and $c\mu_b$, respectively. As soon as the system becomes empty, the server turns off and again starts the service when N customers are accumulated. The two classes of traffic are originated in poisson fashion with arrival rate depending on the server's status. Let i denotes the state of server as

$$i = \begin{cases} 0, & \text{servers are idle} \\ 1, & \text{servers are turned off and rendering service in single service mode} \\ b, & \text{servers are turned on and rendering service in batch service mode} \end{cases}$$

Now state dependent arrival rate is given by

$$\lambda_{i,j} = \begin{cases} \lambda_{0,j}, & \text{arrival rate of } j^{\text{th}} (j = 1,2) \text{ customer when the servers are idle} \\ \lambda_{1,j}, & \text{arrival rate of } j^{\text{th}} (j = 1,2) \text{ customer when the servers are in single service mode} \\ \lambda_{b,j}, & \text{arrival rate of } j^{\text{th}} (j = 1,2) \text{ customer when the servers are in batch service mode} \end{cases}$$

The arrival rates when servers are in turned off state and busy in single mode service are denoted by $\Lambda_0 (= \lambda_{0,1} + \lambda_{0,2})$ and $\Lambda_1 (= \lambda_{1,1})$ respectively. The servers serve the customers in single service mode one by one in FCFS pattern with rate $n\mu_1$. We consider the setup time which is taken by servers after the accumulation of N customers to start the service of the first customer. The setup time is exponentially distributed with mean $\frac{1}{\beta}$. After threshold 'd', the arrival occurs with rate $\Lambda_2 (= \lambda_{b,1})$. When the number of customers reaches a threshold level 'd' of queue size, then the type 2 customers are dropped and the servers serve all the type 1 customers in batch. The total system capacity is K. The state transition diagram for the model is shown in fig 1. We define the following probabilities to construct the birth-death process.

P_{0,n}(t) : Probability that the server is turned off and n customers are present in the system at time t.

P_{1,n}(t) : Probability that the server is turned on and n customers are present in the system at time t.

3. Steady State Equations

We have constructed the following steady state equations with appropriate transition rates (fig.1) are as follows:

$$-\Lambda_0 P_{0,0} + n\mu_1 P_{1,1} + c\mu_0 \sum_{i=d+1}^K P_{1,i} = 0 \quad (1)$$

$$-\Lambda_0 P_{0,n} + \Lambda_0 P_{0,n-1} = 0, 1 \leq n \leq N - 1 \quad (2)$$

$$-(\Lambda_0 + \beta)P_{0,N} + \Lambda_0 P_{0,N-1} = 0 \quad (3)$$

$$-(\Lambda_0 + \beta)P_{0,n} + \Lambda_0 P_{0,n-1} = 0, N + 1 \leq n \leq d - 1 \quad (4)$$

$$-(\Lambda_0 + \beta)P_{0,d} + \Lambda_0 P_{0,d-1} = 0 \quad (5)$$

$$-(\Lambda_0 + \beta)P_{0,n} + \Lambda_0 P_{0,n-1} = 0, d + 1 \leq n \leq K - 1 \quad (6)$$

$$-\beta P_{0,K} + \Lambda_0 P_{0,K-1} = 0 \quad (7)$$

$$-(\Lambda_1 + n\mu_1)P_{1,1} + n\mu_1 P_{1,2} = 0 \quad (8)$$

$$-(\Lambda_1 + n\mu_1)P_{1,1} + \Lambda_1 P_{0,n-1} + n\mu_1 P_{1,n+1} = 0 \quad (9)$$

$$-(\Lambda_1 + N\mu_1)P_{1,N} + \beta P_{0,N} + \Lambda_1 P_{0,N-1} + N\mu_1 P_{1,N+1} = 0 \quad (10)$$

$$-(\Lambda_1 + n\mu_1)P_{1,n} + \beta P_{0,n} + \Lambda_1 P_{0,n-1} + n\mu_1 P_{1,n+1} = 0, N + 1 \leq n \leq d - \quad (11)$$

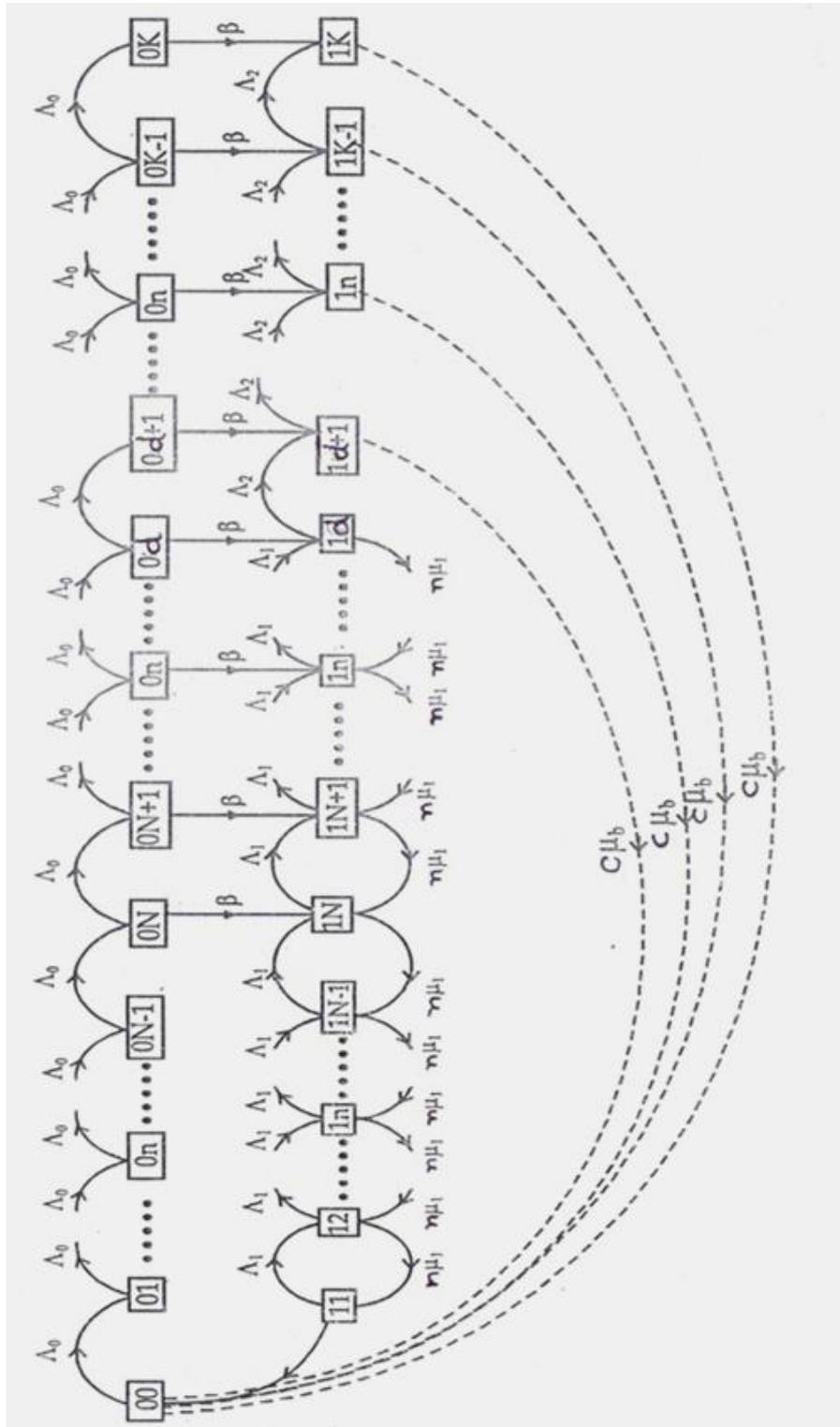


Fig.1 Steadystatediagram

$$-(\Lambda_2 + n\mu_1)P_{1,d} + \beta P_{0,d} + \Lambda_1 P_{0,d-1} = 0 \quad (12)$$

$$-(\Lambda_2 + c\mu_b)P_{1,n} + \beta P_{0,d} + \Lambda_2 P_{1,n-1} = 0, d + 1 \leq n \leq K - 1 \quad (13)$$

$$-c\mu_b P_{1,K} + \beta P_{0,K} + \Lambda_2 P_{1,K-1} = 0 \quad (14)$$

4. Queue Size Distribution

In order to obtain the queue size distribution we apply recursive technique for solving these equations.

Using equation (2), we have

$$P_{0,n} = P_{0,n-1} = P_{0,0}, 1 \leq n \leq N - 1 \quad (15)$$

From equation (3), we get

$$P_{0,N} = (1 + \gamma)^{-1} P_{0,0} \quad (16)$$

Where $\gamma = \frac{\beta}{\Lambda_0}$

Equation (4), gives

$$P_{0,n} = (1 + \gamma)^{-1} P_{0,n-1}, N + 1 \leq n \leq d - 1 \quad (17)$$

From equation (5), we have

$$P_{0,d-1} = (1 + \gamma) P_{0,d} \quad (18)$$

Equation (6), (7) and (8) gives respectively

$$P_{0,d-1} = (1 + \gamma) P_{0,d}, d + 1 \leq n \leq K - 1 \quad (19)$$

$$P_{0,K-1} = \frac{\beta}{\Lambda_0} P_{0,K} \quad (20)$$

$$P_{1,2} = (1 + \rho_1) P_{1,1} \quad (21)$$

Where $\rho_1 = \frac{\Lambda_1}{n\mu_1}$

Now substituting the value of n=2 in equation (9) and by using the equation (21), we get

$$P_{1,n} = \left(\frac{1 - \rho_1^n}{1 - \rho_1} \right) P_{1,1}, 2 \leq n \leq N \quad (22)$$

Where $\rho_1 = \frac{\Lambda_1}{n\mu_1}$

Combining equation (10) and (11), we have

$$P_{1,n} = \left(\frac{1-\rho_1^n}{1-\rho_1} \right) P_{1,1} - \frac{\beta}{n\mu_1} P_{0,0} \left[\sum_{i=1}^{n-N} \left(\frac{1-\rho_1^{n-i-N+1}}{1-\rho_1} \right) v^i \right], N+1 \leq n \leq d-1 \quad (23)$$

Where $v = \frac{\Lambda_0}{\Lambda_0+\beta}$

From equation (12), we get

$$P_{1,d} = \alpha P_{1,d-1} + \alpha_1 P_{0,d} \quad (24)$$

Where $\alpha = \frac{\Lambda_1}{\Lambda_2+n\mu_1}$, and $\alpha_1 = \frac{\beta}{\Lambda_2+n\mu_1}$

Using equation (13), we get

$$P_{1,n} = \alpha_2^{n-d} \cdot \alpha P_{1,d-1} + [\alpha_2^{n-d} \cdot \alpha_1 + \alpha_3 \sum_{i=1}^{n-d} \alpha_2^{n-d-i} \cdot \sigma^i] P_{0,d}, d+1 \leq n \leq K-1 \quad (25)$$

Where $\sigma = \frac{\Lambda_0}{\Lambda_0+\beta}$ and $\alpha_3 = \frac{\beta}{\Lambda_2+c\mu_b}$

From equation (14) and (25), we get

$$P_{1,K} = \frac{\Lambda_2}{c\mu_b} \left[\alpha_2^{K-d-1} \cdot \alpha P_{1,d-1} + \left\{ \alpha_2^{K-d-1} \cdot \alpha_1 + \alpha_3 \sum_{i=1}^{K-d-1} \alpha_2^{K-d-1-i} \sigma^i \right\} P_{0,d} \right] + \frac{\Lambda_0}{c\mu_b} P_{0,K-1} \quad (26)$$

From equation (1), we have

$$P_{1,1} = \frac{\Lambda_0}{n\mu_1} P_{0,0} - \frac{c\mu_b}{n\mu_1} \sum_{i=d+1}^{K-1} P_{1,i} - \frac{c\mu_b}{n\mu_1} \left[\frac{\Lambda_2}{c\mu_b} [\alpha_2^{K-d-1} \cdot \alpha P_{1,d-1} + \{ \alpha_2^{K-d-1} \cdot \alpha_1 + \alpha_3 \sum_{i=1}^{K-d-1} \alpha_2^{K-d-1-i} \sigma^i \} P_{0,d} + \Lambda_0 c\mu_b P_{0,K-1}] \right] \quad (27)$$

Now equation (15)-(22) give the steady state probabilities depending on $P_{0,0}$ which is computed by using normalizing condition given by

$$\sum_{n=0}^K P_{0,n} + \sum_{n=1}^K P_{1,n} = 1 \quad (28)$$

5. Performance Measures

We obtain performance measures for N-policy finite capacity queueing system with integrated traffic and setup time with help of steady state probabilities.

The expected number of customer in the queue

$$E(Q) = \sum_{n=0}^K (n-1)P_{0,n} + \sum_{n=1}^K (n-1)P_{1,n} \quad (29)$$

The expected number of customers in the system is obtained by

$$E(N) = \sum_{n=0}^K nP_{0,n} + \sum_{n=1}^K nP_{1,n} \quad (30)$$

The probability of the server being turned off (excluding setup state) is

$$P(O) = \sum_{n=0}^{N-1} P_{0,n} \quad (31)$$

The probability of the server being under setup is

$$P(S) = \sum_{n=N}^K P_{0,n} \quad (32)$$

The probability of the server being idle (including turned off state and setup state)

$$P(I) = P(O) + P(B) \quad (33)$$

The probability of the server being busy is

$$P(B) = \sum_{n=1}^K P_{1,n} \quad (34)$$

Applying the memory less property of the Poisson process, the length of the idle period is sum of N exponential random variables, each having mean rate $\frac{1}{\Lambda_0}$ so that

$$E(I) = \frac{N}{\Lambda_0} \quad (35)$$

In this model busy period and idle period generate an alternative renewal process, hence

$$\frac{E(B)}{E(I)} = \frac{1-P(I)}{P(I)} \quad (36)$$

Where P(I) is computed in equation (33). Now, with the help of equation (31) and (34), we get

the expected busy period of the server as

$$E(B) = \left(\frac{1-NP_{0,0}}{NP_{0,0}+P(S)} \right) \left(\frac{N}{\Lambda_0} \right) \quad (37)$$

6. Conclusion

In this investigation, we studied a finite capacity prioritized queueing system with setup time along with two classes of customers, which are served singly as well as batches. We used recursive technique to obtain the queue size distribution. The M/M/C/K queueing model investigated in our study has wide range of applications in real world situations of telecommunications wherein resources such as transmission lines and switching nodes are shared by mixed type traffic. These situations may arise in which non priority customers may be rejected whereas priority class customers are served in batch after a threshold level of the number of customers. Thus it is required to evaluate the optimal threshold policy by using performance measures. Our study provides a guideline to the system engineers in designing more efficient system based optimal control policy.

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